Lesson Notes

**Introduction**

Find the surface area and volume for each of the following 3-D objects.

**a) sphere**

![Sphere Diagram]

- **Surface Area Formula**
  \[ SA = 4\pi r^2 \]
  \[ = 4\pi \times 8^2 \]
  \[ = 804.25 \text{ cm}^2 \]

- **Volume Formula**
  \[ V = \frac{4}{3}\pi r^3 \]
  \[ = \frac{4\pi \times 8^3}{3} \]
  \[ = 2144.66 \text{ cm}^3 \]

**b) rectangular prism**

![Rectangular Prism Diagram]

- **Surface Area Formula**
  \[ SA = 2lw + lh + wh \]
  \[ = 2\times 9 \times 12 + 3 \times 9 + 3 \times 12 \]
  \[ = 342 \text{ in}^2 \]

- **Volume Formula**
  \[ V = l \times w \times h \]
  \[ = 9 \times 12 \times 3 \]
  \[ = 324 \text{ in}^3 \]
c) square pyramid

\[ V = \frac{1}{3} \times \text{base area} \times h \]

\[ V = \frac{64}{3} \times 5.75 = 122.67 \text{ cm}^3 \]

<table>
<thead>
<tr>
<th>Base area</th>
<th>Triangle area</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 cm \times 8 cm = 64 cm ^2</td>
<td>7 \times 8 \times \frac{1}{2} = 28 cm ^2</td>
</tr>
<tr>
<td>\text{Total} = 176 cm ^2</td>
<td></td>
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\[ A_{\text{base}} = 7 \times 8 \times 4 = 112 \text{ cm}^2 \]

\[ A_{\text{triangle}} = \frac{7 \times 8}{2} = 28 \text{ cm}^2 \]

\[ A_{\text{total}} = 176 \text{ cm}^2 \]

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d) rectangular pyramid

\[ V = \frac{1}{3} \times \text{base area} \times h \]

\[ 192 \times 8 = 512 \text{ cm}^3 \]

<table>
<thead>
<tr>
<th>Base area</th>
<th>Area of 4 triangles</th>
</tr>
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<tbody>
<tr>
<td>12 cm \times 16 cm = 192 cm ^2</td>
<td>\frac{12 \times 16}{2} \times 2 = 160 \text{ cm}^2</td>
</tr>
<tr>
<td>\text{Total} = 487.6 cm ^2</td>
<td>16 \times 10 = 160 \text{ cm}^2</td>
</tr>
<tr>
<td>\text{Total} = 13516 \text{ cm}^2</td>
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e) right cylinder

Volume Formula

\[ V = \text{area of end} \times \text{height} \]

\[ = 78.5 \times 12 \]
\[ = 942 \text{ ft}^3 \]

Total area = 78.5 + 78.5 + 376.8
\[ \text{total area} = 533.8 \text{ ft}^2 \]

Surface Area Formula

\[ \pi r^2 \]

\[ = \pi \times 5^2 = 78.5 \]

\[ c = \pi d \]
\[ = \frac{31.4}{12} = 31.4 \]

\[ A = 376.8 \]

f) right cone

Volume Formula

\[ V = \frac{1}{3} \pi r^2 h \]

\[ \text{area of base} \times h \]
\[ \frac{153.9 \times 18}{3} \]
\[ = 923.4 \text{ m}^3 \]

Surface Area Formula

\[ \pi r^2 + \pi r l \]

\[ = (\pi \times 7 \times 19.3) + (\pi \times 7^2) \]
\[ = (424.4 + 153.9) \]
\[ = 578.3 \text{ m}^2 \]
Example 1

Find the unknown measurement in each of the following:

a) a sphere

\[ A = 4\pi r^2 \]
\[ V = \frac{4}{3} \pi r^3 \]

i) Use the surface area formula to solve for the radius.

\[
\frac{4536.46}{4\pi} = 4536.46 \div 4\pi = r^2 \\
\sqrt{361} = r \\
r = 19
\]

ii) Use the volume formula to solve for the radius.

\[
\frac{4\pi r^3}{3} = \frac{28731}{4\pi} \\
4\pi r^3 = 86193 \\
r^3 = 86193 \\
r = \sqrt[3]{86193} \\
r = 9.29 \text{ (approx)}
\]

SA = 4536.46 cm²
V = 28731 cm³

b) right cone

\[ A = \pi r s + \pi r^2 \]
\[ V = \frac{1}{3} \pi r^2 h \]

i) Use the surface area formula to solve for the slant height.

\[
320.44 = (\pi \times 6 \times 3) + (\pi \times 6^2) \\
320.44 = 18.8496 \times 9 + 113.097 \\
320.44 - 113.097 = 18.8496 \times s \\
207.343 = 18.8496 \times s \\
\frac{207.343}{18.8496} = s \\
s = 11
\]

ii) Use the volume formula to solve for the height.

\[
347.57 = \frac{\pi \times 6^2 \times h}{3} \\
\frac{347.57}{18.8496} = \frac{\pi \times 36 \times h}{3} \\
\frac{347.57}{18.8496} = \frac{1042.71}{18.8496} \\
10.99986 = s \\
s = 11
\]

SA = 320.44 m²
V = 347.57 m³
Example 2  Sketch each shape and determine the indicated quantity.

a) A square pyramid has a base measuring 5 ft. by 5 ft. The height of the pyramid, from the centre of the base to the apex is 7 ft. Calculate the surface area of the pyramid.

\[ SA = \text{base area} + 4 \times \text{triangle area} \]
\[ 5 \times 5 \]

b) A cylindrical water tank with an open top has a volume of 5702 m\(^3\) and a radius of 11 m. Calculate the height of the tank.

\[ V = \pi r^2 h \]
\[ \frac{5702}{\pi \times 11^2} = 15 \]

\[ 15 = h \]
Example 3 Find the surface area and volume of this 3-D object.

a) surface area

1. **Cone with top**
   
   \[ SA = \pi r s = \pi \times 6 \times 13.4 \]
   
   \[ = 252.6 \text{ cm}^2 \]

2. **Sphere**
   
   \[ \text{SA} = \frac{4\pi r^2}{2} = \frac{4\pi \times 6^2}{2} \]
   
   \[ = 226.2 \text{ cm}^2 \]

   \[ \text{Total surface area} = 252.6 + 226.2 = 478.8 \text{ cm}^2 \]

b) volume

1. **Cone**
   
   \[ V = \frac{1}{3} \pi r^2 h = \frac{\pi r^2 h}{3} \]
   
   \[ = \frac{\pi \times 6^2 \times 12}{3} \]
   
   \[ = 452.4 \text{ cm}^3 \]

2. **Sphere**
   
   \[ V = \frac{4}{3} \pi r^3 \]
   
   \[ = \frac{4 \times \pi \times 6^3}{3} \]
   
   \[ = 904.8 \text{ cm}^3 \]

\[ \text{Total volume} = 452.4 + 904.8 = 1357.2 \text{ cm}^3 \]

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Example 4

Find the surface area and volume of this 3-D object.

a) surface area

2 of these

b) volume

cylinder

\[ V = \pi \times 10^2 \times 4.5 \]

\[ = 1413.7 \times 2 \text{ ends} \]

\[ = 2827.4 \]

handle

\[ V = \pi \times 1.5^2 \times 6 \]

\[ = 42.4 \]

Total Vol = 2827.4 + 42.4

\[ = 2869.8 \text{ cm}^3 \]

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Example 5  Find the surface area and volume of this 3-D object.

a) surface area

b) volume

\[ V = \text{Volume of block} = 20 \times 20 \times 60 \text{ mm} = 24000 \text{ mm}^3 \]

\[ V = 24000 - 3141.6 \\ - 3141.6 \text{ mm}^3 \\ = 20858.4 \text{ mm}^3 \]

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Example 6

Find the surface area and volume of this 3-D object.

a) surface area

b) volume

Vol of base = $5 \times 30 \times 20 = 3000 \text{ cm}^3$

Vol of square pyramid = $\frac{1}{3} \times \text{ base area} \times h$

Vol of rectangular pyramid = $\frac{1}{3} \times \text{ base area} \times h$

$600 \times 15 = 9000 \text{ cm}^3$

Total vol = $3000 + 9000 = 12000 \text{ cm}^3$
Example 7  Find the surface area and volume of this 3-D object.

a) surface area

\[
a = \sqrt{(14^2 - 10^2)} = 9.8
\]

\[
V_{\text{shaded}} = \frac{\pi \times 12^2 	imes 9.8}{3} = 1026.3\ m^3
\]

b) volume

\[
a = \sqrt{(5^2 - 3^2)} = 4
\]

\[
V_{\text{small}} = \frac{\pi \times 3^2 \times 4}{3} = 37.7\ m^3
\]