Grade 11 Essential Math

Relations and Patterns
The cost of hiring a roofer varies directly with the time needed to shingle a roof. If a roofer earns $1000 for a job that takes 40 hours:

a) Determine the constant of variation (fancy word for how the roofer earns per hour).

\[ \text{rate per hour} \times \text{time} = \text{cost} \]

What do you multiply time by to get cost?

\[ \text{rate per hour} \]

b) Write a formula for this problem. Use \( C \) for cost and \( h \) for hours.

\[ \frac{1000}{40} = \frac{25}{1} \text{ constant} \]

\[ C = \$25 \times h \]

Where \( C \) is cost \( h \) is hours

c) Find the cost of hiring a roofer if the job will last 4 hours. Use your formula.

\[ C = 25 \times 4 = \$100 \]

d) A roofing job costs $500. How long did the job last? Use your formula.

\[ \frac{500}{25} = \frac{25}{25} \]

\[ h = 20 \text{ hours} \]

e) Identify the dependent and independent variables in this problem.

- Variables are \( C \) and \( h \)
- Constant is $25
- \( \text{Cost depends on hours} \)
- \( \text{hours depend on cost} \)
Example 2

Bob's wages vary directly as the number of hours that he works. In other words, the more he works, the more he gets paid. Bob's wages for 5 hours are $51.25.

a) Determine the constant of variation (fancy word for hourly pay).

\[ \frac{51.25}{5} = \$10.25 \]

b) Write out a formula for this problem. Use \( W \) for wages and \( h \) for hours.

\[ W = 10.25 \times h \]

c) How much will Bob get paid if he works for 30 hours? Use your formula.

\[ W = 10.25 \times 30 = \$307.50 \]

d) Bob got paid $35.88. How many hours did he work? Use your formula.

\[ 35.88 = 10.25 \times h \]
\[ h = 3.5 \text{ hours} \]

f) Identify the dependent and independent variables in this problem.

Wages depend on hours worked.
Example 3
To rent a canoe at the beach, a rental company charges a flat fee of $20 plus $5 per hour.

a) Fill in the table of values below:

\[
\text{Charge} = 20 + 5h
\]

<table>
<thead>
<tr>
<th>Hours</th>
<th>Rental Charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$25</td>
</tr>
<tr>
<td>2</td>
<td>$30</td>
</tr>
<tr>
<td>3</td>
<td>$35</td>
</tr>
</tbody>
</table>

b) Sketch a graph of this equation. Hint: Label the x and y axis and two points on the line.

c) Write a formula for this rental company. Use C for cost, and h for hours.

\[ C = 20 + 5h \]

d) Calculate the cost of renting a canoe for 8 hours. Use your formula.

\[
C = 20 + 5 \times 8 \\
= 60
\]
Example 4
An appliance repair person charges a flat fee of $30 to make a service call plus $23 per hour.

a) Fill in the table of values below:

<table>
<thead>
<tr>
<th>Hours</th>
<th>Service Charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>$53</td>
</tr>
<tr>
<td>1</td>
<td>$76</td>
</tr>
</tbody>
</table>

\[ C = 30 + 23 \times \text{Hours} \]

b) Write a formula for this appliance repair person. Use \( C \) for charge, and \( h \) for hours.

\[ C = 30 + 23h \]

c) Calculate the cost of a repair that lasts 3.5 hours.

\[ C = 30 + (23 \times 3.5) = 110.50 \]
Example 5
Raymond is installing a 20 foot drain pipe so he can drain the water far away from his house. He needs the drain pipe to drop 1 in. for every 4 feet of length so the water will drain properly.

a) Draw a sketch.

b) Calculate the slope of the drain pipe. Express it in a percent.
   Hint: Convert 4 feet to inches.
Essential Math 305 — Relations and Formulas — Miscellaneous Questions

Example 6
Your uncle Milo wants you to build some steps down to the beach at his cabin. He has measured his foot and it is 14 inches long! He wants the steps to have a 70% grade. Assume you want a run of at least 14 inches for the steps. What kind of rise will you need to maintain the 70% grade?

a) Draw a sketch.

\[ \frac{70\%}{100} = \frac{\text{rise}}{\text{run}} \]
\[ = \frac{7}{10} = \frac{70}{100} \]
\[ \frac{\text{rise}}{\text{run}} = \frac{7}{10} = \frac{x}{14} \]

b) Calculate the rise of an individual step.

almost 9.8 inches!

10 inches!
Example 7
Jim is training for the Skills Canada Roofing competition. He wants to train for the competition by building a ramp with a 6/12 pitch and then running up and down the ramp carrying shingles. If he has 30 feet of horizontal distance to build this ramp, how high will the ramp be?

\[
\frac{\text{Rise}}{\text{Run}} = \frac{6}{12} = \frac{x}{30}
\]

\[x = 15\]

Example 8
Barry is a carpenter. He sends his helper out to measure the slope of some rafters they are going to install. Barry’s helper says the slope is 0.83. Express this slope as a pitch. If the rafters are 28 feet long from end to end, what will height of the rafter be?

\[
\text{Pitch} = \frac{x}{12} = \frac{83}{100}
\]

\[
\frac{10}{14} = \frac{x}{12} \Rightarrow \frac{9.96}{12}
\]
1. Scott is driving a van full of snowboarders from Altona, Manitoba to Fernie, British Columbia. A sign says that Fernie is 1450 kilometres miles away. Note that 1 km is 0.621 miles.
   a) Scott's van odometer is in miles. How many miles is 1450 kilometres?

\[
\frac{\text{km}}{\text{miles}} \times \frac{1}{0.621} = 1450
\]

b) If Scott drives a steady 100 km/h the entire distance, how many hours and minutes will it take him to get to Fernie?

\[
\frac{\text{km}}{\text{hours}} \times \frac{180}{1} = \frac{1450}{x}
\]

2. Mackenzie mentions to her friends that she took the bus to Minneapolis, Minnesota from Altona. The bus driver made the trip in 5 hours! The distance from Altona to Minneapolis is 663 kilometers.
   a) What was the speed of the bus?

\[
\frac{\text{km}}{\text{hour}} \times \frac{663}{5} = \frac{132.6}{1}
\]

b) The speed limit in the USA is 120 km/h. Approximately how much time did the bus driver save by driving faster than the speed limit?

\[
\text{going } 120 \text{ km/h for 663 km.}
\]

\[
\text{what } \begin{array}{c} \text{true} \\ \text{solve} \end{array} \text{ extra }
\]

\[
\frac{\text{km}}{\text{hours}} \times \frac{120}{663} = \frac{x}{5.625}
\]

\[
60 \text{ mins } \times 0.525
\]

\[
= 31.5 \text{ mins saved}
\]
3. Sage and Zack get summer jobs herding sheep near Gimli. On his first day, Sage earns $92 for 8 hours of work.

a) Calculate Sage's hourly wage.

\[
\text{Pay} \quad \frac{92}{8} = \frac{11.5}{1}
\]

b) Write a formula that Sage can use to figure out his pay with any number of hours.

\[
\text{Pay} = \$11.50 \times \text{hours}
\]

4. Zack's boss notices that he has trouble getting to work on time in the morning. He gives him a bonus of $15 if he gets to work early but takes $20 off his daily pay if he arrives late.

a) Assuming Zack arrives early and works 8 hours, calculate his pay per hour.

\[
\$92 + 15 = \$107 \text{ for 8 hours}
\]

\[
107 \div 8 = \$13.38 \text{ per hour}
\]

b) Assuming Zack arrives late and works 8 hours, calculate his pay per hour.

\[
\$92 - 20 = \$72 \text{ for 8 hours}
\]

\[
72 \div 8 = \$9.00 \text{ per hour}
\]
5. Adam is building a garage for his neighbour. The roof rises 3 feet for every 5 feet of width.

a) Express the garage roof slope as a percent.

\[
\frac{\text{rise}}{\text{run}} = \frac{3}{5} = \frac{60}{100} = 60\%
\]

b) Express the garage roof slope as a pitch.

\[
\frac{3}{5} = \frac{x}{12}
\]

approx \[ \frac{7.2}{\text{pitch}} \]

6. Daniel went hiking during semester break and mentions to his friends that he climbed a hill that had a slope of 0.667.

Express the slope as a grade.

\[
\frac{\text{Rise 66.7}}{\text{Run 100}} \quad \text{make into } \frac{x}{100} \quad \frac{66.7}{100} = \frac{x}{12}
\]

7. Scott climbed the same hill last year and said it is equivalent to a roof with an 8/12 pitch. Prove that Daniel and Scott climbed are describing the same slope.

Express the slope as a pitch.

\[
\frac{\text{rise}}{\text{run}} = \frac{8}{12}
\]
8. Dusty is mixing oil and gasoline for his uncle's antique chainsaw. The chainsaw needs 1 part oil for every 15 parts gasoline.
   
a) If Dusty has 500 mL of oil, how many litres of gasoline will he need?
   \[
   \text{oil} \quad \frac{1}{15} = \frac{500 \text{ mL}}{7500 \text{ mL}} = 0.0667 \text{ L} = 0.07 \text{ L}
   \]
   
b) If Dusty has a 5 litre container of gasoline, how much oil should he add?
   \[
   \text{oil} \quad \frac{1}{15} = \frac{333 \text{ mL}}{5000 \text{ mL}}
   \]

9. Nathan is filling his hot tub with water. He reads the capacity on the side of the hot tub and it says 1370 litres. He estimates it took his garden hose 2.5 hours to fill the tub.
   
a) What is the flow rate of his garden hose per hour?
   \[
   \frac{\text{Litres}}{\text{Hour}} = \frac{1370}{2.5} = 548 \text{ L/hr}
   \]
   
b) If Nathan buys a larger hot tub (3270 litres), how much time will it take to fill it?
   \[
   \frac{548}{1} = \frac{3270}{\text{approx. 6 hr}} = 5.97 \text{ hours}
   \]
10. Nat has no money in his bank account and needs to replace two tires on his truck today before he can go hunting. He gets paid in two weeks. Payday Loan charges a fee of $25 for every $100 borrowed. After two weeks, you must pay back what you owe plus the fee.

a) If Nat borrows $350 to pay for the tires, how much will he need to pay back in two weeks?

\[
\frac{25}{100} = \frac{x}{350}
\]

\[
x = \frac{350 \times 25}{100} = 87.50
\]

\[
\text{Total amount paid back} = 87.50 + 350 = 437.50
\]

b) What percent interest (per year) did Nat pay when borrowing money from Payday Loan?

\[
I = \frac{25 \times (2)}{100 \times (52)} = 6.50\%
\]

11. Jesse is helping a carpenter install a new slide at the park. The slide is 20 feet long and is supposed to have a slope of 30 degrees.

How high should Jesse build the ladder for this slide?

\[
\sin 30^\circ = \frac{\text{opp}}{20} = 10 \text{ ft}
\]
12. Mason is installing chrome sidepipes on his car. They need to be mounted with a 1 inch drop for every 2 feet of length so that condensation will drain properly.

a) Express the slope as a fraction.

\[
\frac{\text{rise}}{\text{run}} = \frac{1}{24}
\]

b) Express the slope as a percentage.

\[
\frac{1}{24} \times 100\% = 4.17\%
\]

c) If the sidepipes are 56 inches long, how much lower will they be from one end to the other end?

\[
\frac{\text{drop}}{\text{length}} = \frac{x}{56\text{ in}}
\]

\[
2.33\text{ inches lower}
\]