Key

LESSON 1: RATIOS AND PROPORTIONS

What Is a Ratio?

- A ratio is a comparison of one quantity to another.
- A ratio can be expressed as a fraction.
- The numerator and denominator of the fraction are called the terms of the ratio.

Before continuing in this booklet, it would be a good idea to review your cross multiplying and dividing skills. Ask your teacher for the handouts.

PRACTICE: PROPORTIONS AND RATIOS

1. Find the missing term in each of the following proportions.

a) $\frac{3}{5} = \frac{x}{20}$ $20 \times 3 \div 5 = \underline{12}$

b) $\frac{5}{7} = \frac{25}{x}$ $25 \times 7 \div 5 = \underline{35}$

c) $\frac{3}{5} = \frac{x}{20}$ $20 \times 3 \div 5 = \underline{12}$

2. Calculate what percent 3200 is out of 5000. Set up as a proportion.

$\frac{3200}{5000} = \frac{x}{100}$ $\underline{64\%}$

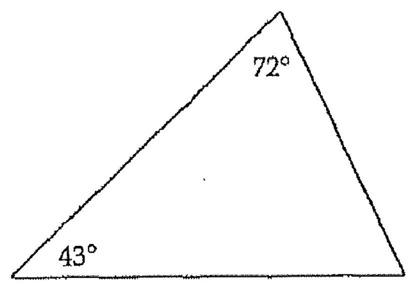
LESSON 2: SUM OF ANGLES = 180

The measures of the three angles of <u>any</u> triangle always add up to a total of 180 degrees (180°). If you know the measures of two of the angles, the third can be easily found.

Note: If a triangle has a box thing in the corner, that angle is 90 degrees. The triangle is then a right triangle.

Example 1

Find the measure of the third angle in the following triangle.

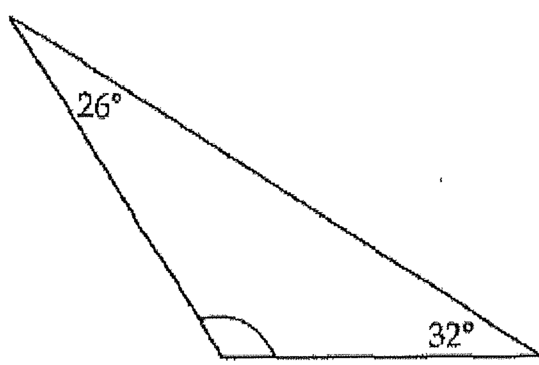


180
−72
−43

65°

Example 2

Find the measure of the missing angle.



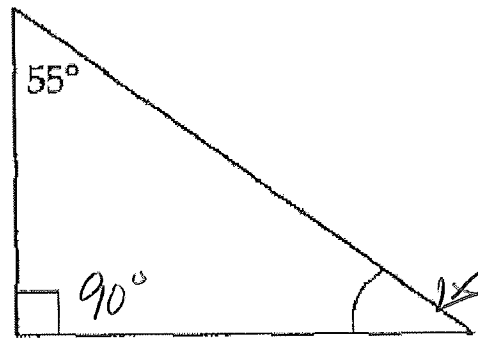
180 − 26 − 32

= 122°

PRACTICE: SUM OF ANGLES

1. Find the measure of the missing angles.

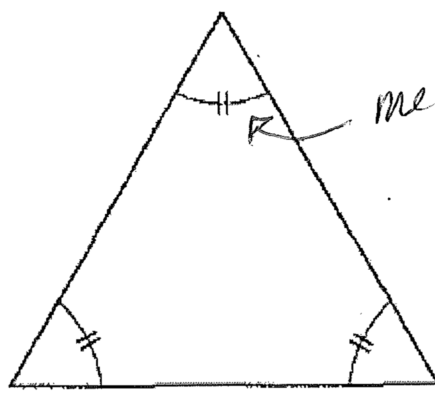
a)



$180 - 55 - 90$

$= \underline{\underline{35°}}$

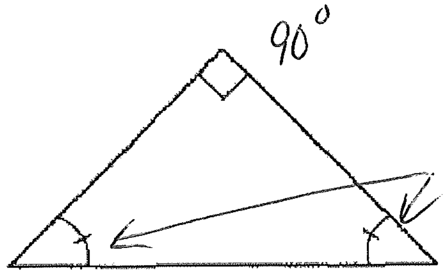
b)



means "same" or "equal"

$180 \div 3 = \underline{60°}$

c)



same. $180 - 90 = 90°$

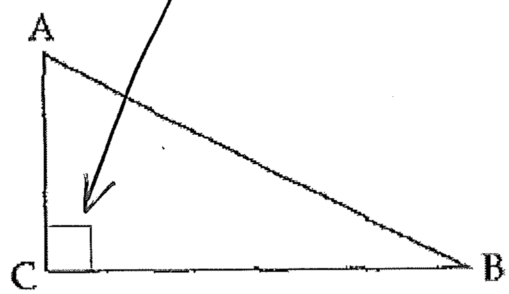
$90 \div 2 = \underline{\underline{45°}}$

LESSON 3: PYTHAGORAS AND RIGHT TRIANGLES

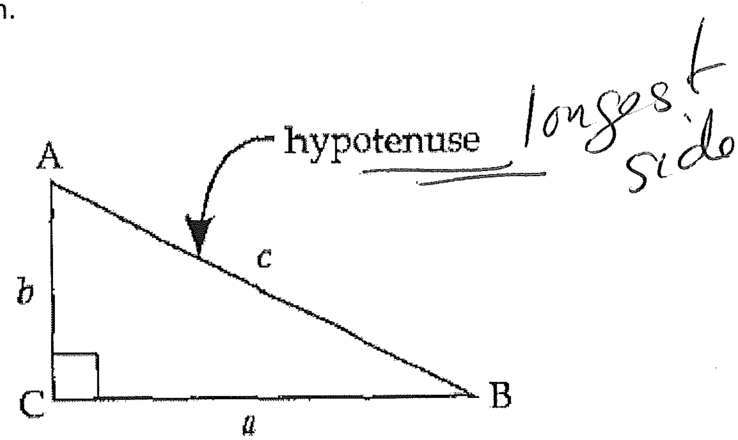
Pythagorean Theorem

Pythagoras lived in Greece around 500 BC. He was a brilliant mathematician, musician, and physicist. He saw mathematical precision in music and astronomy. He believed that music and the orbits of planets around stars could all be explained by mathematical patterns and equations. One discovery credited to and named for him is the Pythagorean theorem.

Right triangles are triangles that have a 90° angle, indicated by a small box in the corner.



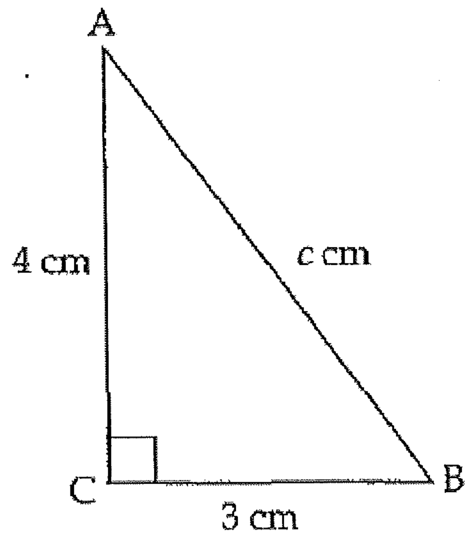
The "hypotenuse" is the longest side opposite the right angle. The shorter two sides are called the "legs" of the right triangle. As before, the sides are labelled in lower case letters according to the angles opposite them.



Example 1

Given the following right triangle, find the length of the hypotenuse.

use posters



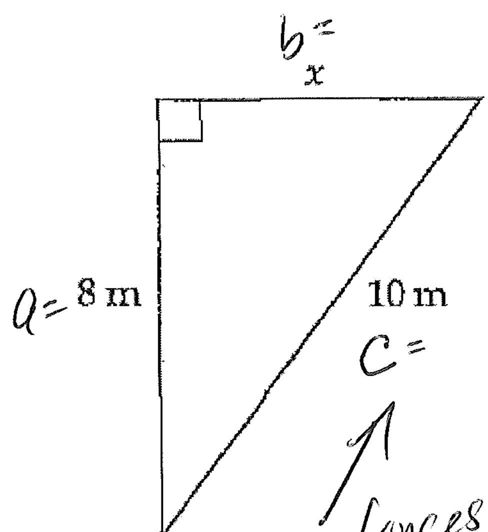
$c = \sqrt{(a^2 + b^2)}$

$c = \sqrt{(4^2 + 3^2)}$

$c = \underline{\underline{5}}$

Example 2

Find the value of x.



longest side

$b = \sqrt{(c^2 - a^2)}$

$b = \sqrt{(10^2 - 8^2)}$

$b = \underline{\underline{6}}$.

Proving an Angle in a Triangle is 90°

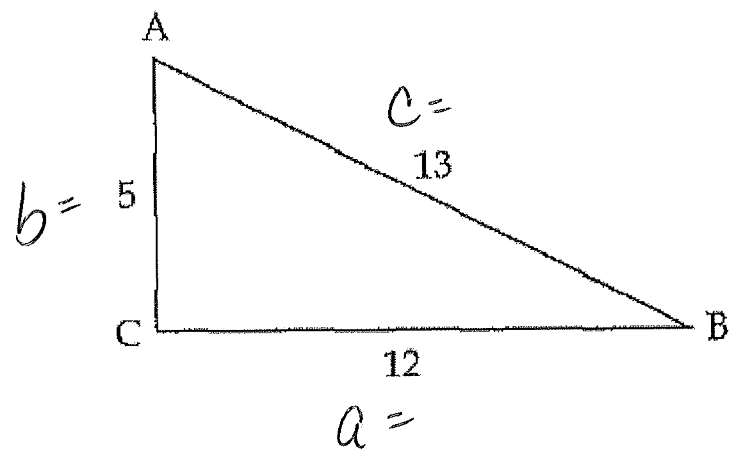
The formula for the Pythagorean theorem works for right triangles. What if you didn't know if the triangle was right-angled? Pythagoras was able to determine that this law is also true. A carpenter or builder uses this law to ensure that two walls in the structure that is being built are at right angles to each other.

How does this work?

You can use the same equation to prove whether or not a triangle has a right angle. Substitute values into the formula for a, b, and c, and simplify. If the left-hand side of the equation equals the right-hand side, then by the Pythagorean rule the triangle must be a right triangle.

Example 1

Determine if the triangle below is a right triangle.



$$c^2 = a^2 + b^2$$
$$13^2 = 12^2 + 5^2$$
$$169 = 144 + 25$$
$$= 169$$

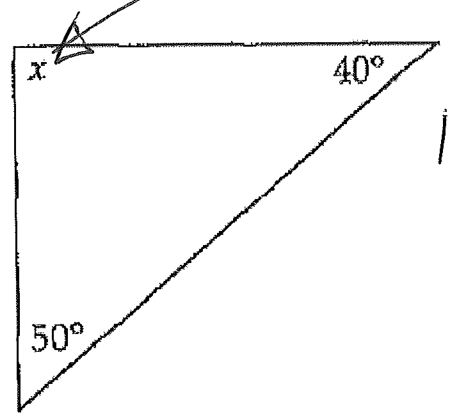
yes! Right △!

Adding the Angles

The three angles in any triangle always add together to make a sum of 180°. Applying this rule to determine if a triangle is a right triangle, if two angles in a triangle add up to 90°, then the third angle must be 90°. Therefore, the triangle must be a right triangle.

Example 1

Is this a right triangle?



$180 - 50 - 40 = 90^\circ$

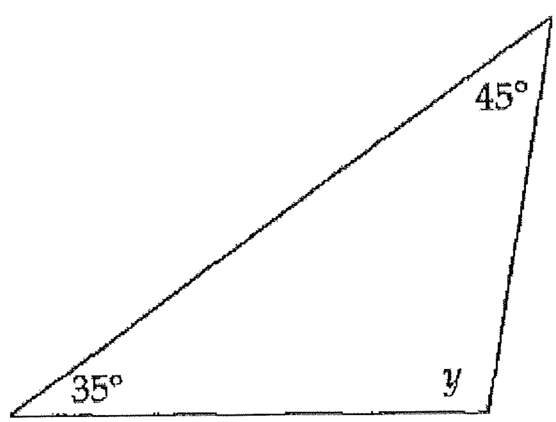
yes!

Example 2

Is this a right triangle?

$180 - 35 - 45 = 100^\circ \neq 90^\circ$

Not a Right $\triangle$!

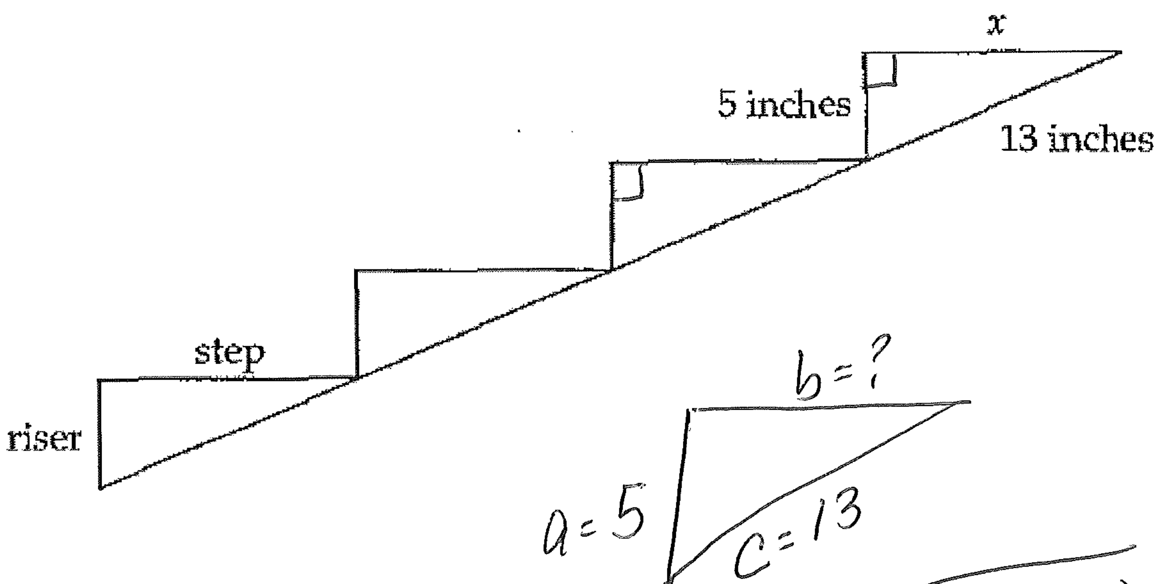


Applying the Pythagorean Theorem to Solving Problems

Identify a triangle having a right angle and identify where the hypotenuse would be.
Then substitute the known values into the formula and solve for the unknown value.

Example 1

You are building a set of stairs. The riser will be 5 inches, and the diagonal support board is 13 inches long for each step. How deep will the step be?



$b = \sqrt{(13^2 - 5^2)}$

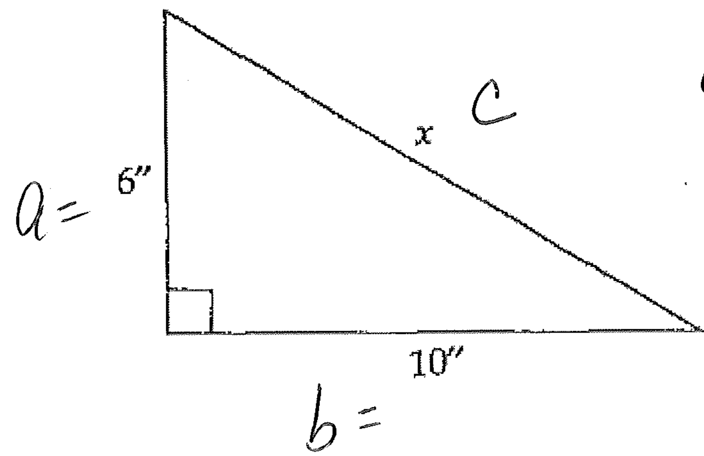
$b = 12$

Stairs are 12 inches deep.

PRACTICE: PYTHAGOREAN THEOREM

1. Find the missing sides. Round off to one decimal place.

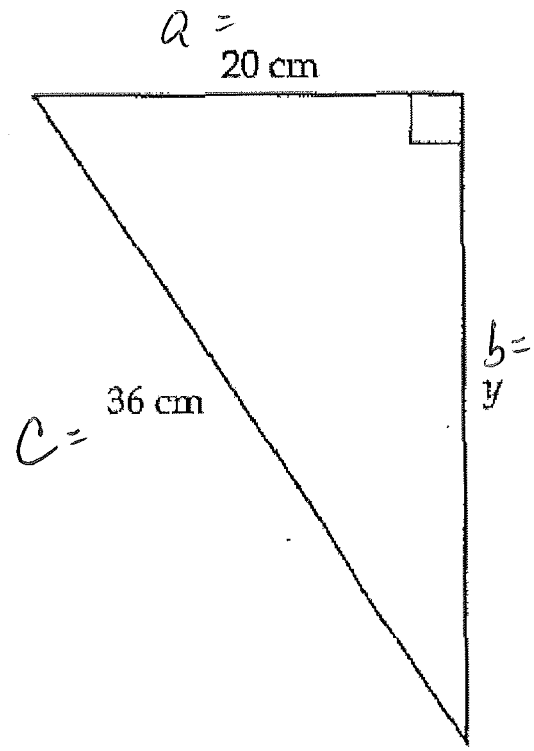
a)



$c = \sqrt{(6^2 + 10^2)}$

$= 11.66''$

b)

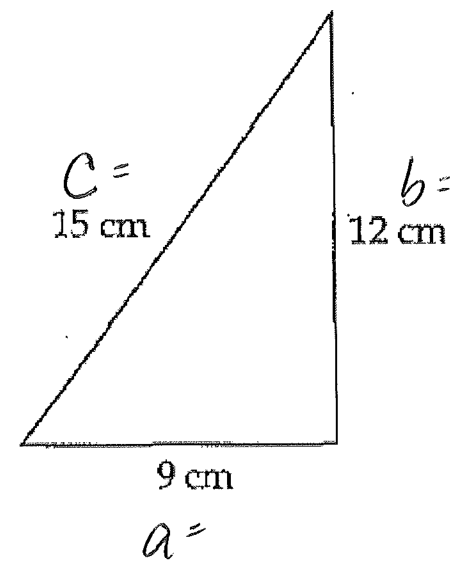


$b = \sqrt{(36^2 - 20^2)}$

$b = 29.93 \text{ cm}.$

2. Use the Pythagorean theorem to prove whether or not these are right triangles.

a)



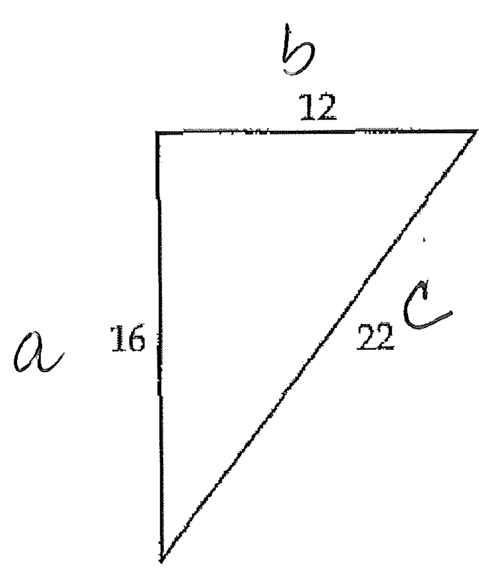
$15^2 = 12^2 + 9^2$

$225 = 144 + 81$

$225 = 225$

yes! Rt Δ.

b)



$22^2 = 16^2 + 12^2$

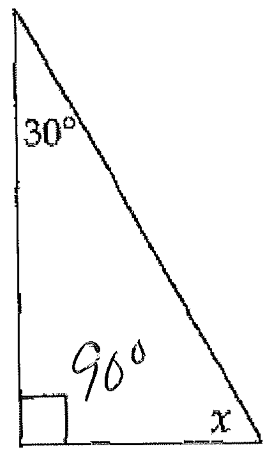
$484 = 256 + 144$

$484 \neq 400$

No! Not a Rt Δ

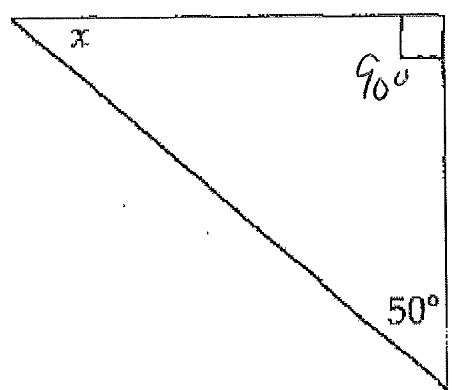
3. Find the missing angle in each triangle.

a)



180 - 90 - 30
= 60°

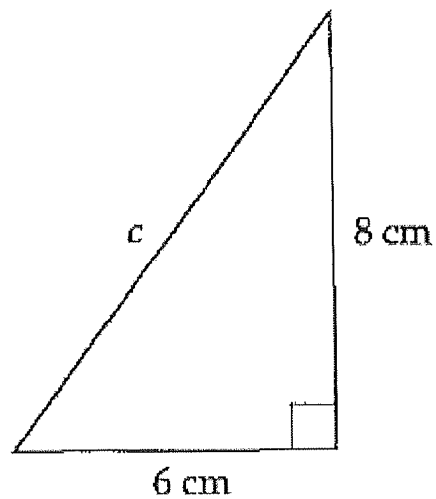
b)



180 - 90 - 50
= 40°

4. Use the Pythagorean relation to find the missing side in each triangle. Show your calculations for full marks.

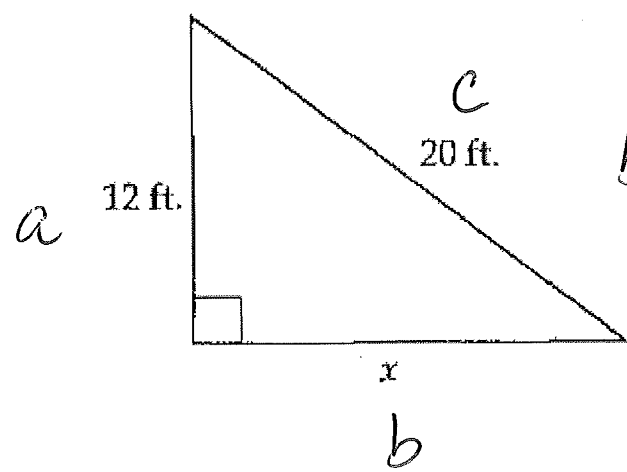
a)



$c = \sqrt{(6^2+8^2)}$

$c = \underline{10 \text{ cm}}$

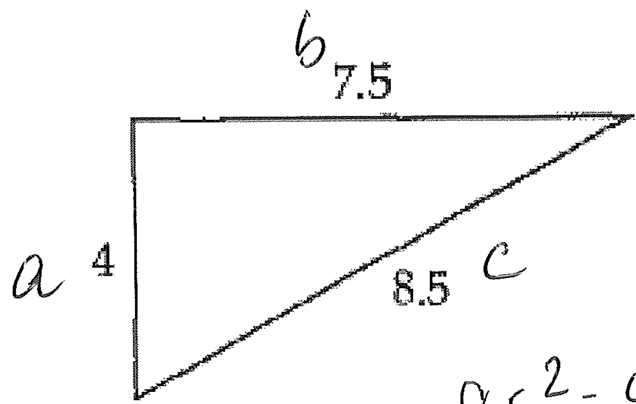
b)



$b = \sqrt{20^2 - 12^2}$

$b = \underline{16 \text{ ft}}$

5. Use the Pythagorean theorem to prove that the given triangle is a right triangle.



$8.5^2 = 4^2 + 7.5^2$

$72.25 = 16 + 56.25$

$72.25 = 72.25$

yes! Rt Δ.

6. Given a right triangle with hypotenuse c and legs a and b.

a) Find c if a = 5 and b = 12

$c = \sqrt{(5^2 + 12^2)}$

$c = \underline{\underline{13}}$

b) Find a if b = 8 and c = 12.

$a = \sqrt{(12^2 - 8^2)}$

$a = \underline{\underline{8.94}}$

LESSON 4: TANGENT RATIO

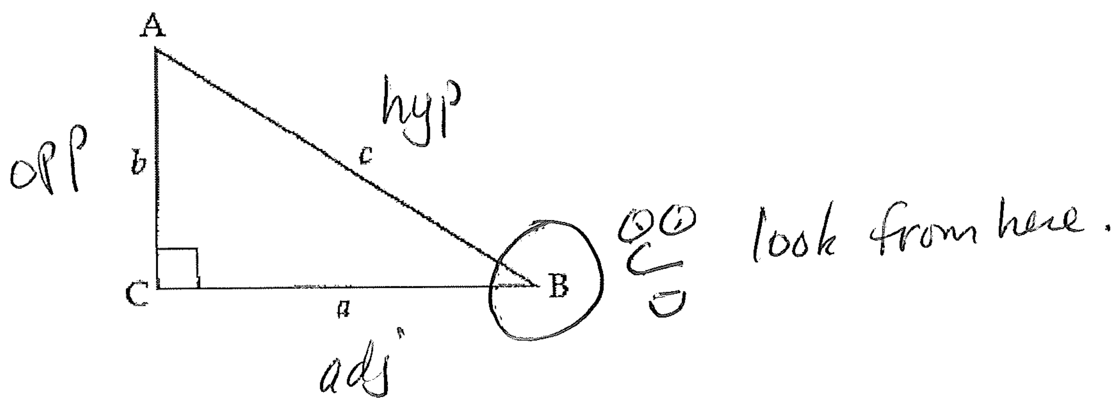
To use trigonometric ratios, you must be able to identify the sides of the triangle from the viewpoint of a designated angle within the triangle. You use three terms, and you must fully understand how to identify them to be successful in this lesson. The right angle is not one of the designated angles; just one of the two smaller angles can be one.

Given a triangle, the three terms are as follows.

- hypotenuse: the side opposite the right angle
- opposite: the side opposite the featured angle
- adjacent: the side next to, or beside, the featured angle

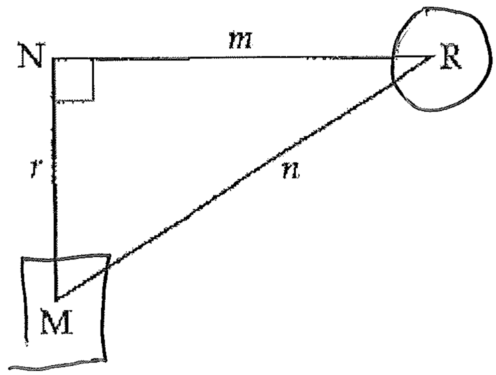
Example 1

Choose ∠B as the designated angle. Identify the hypotenuse, the opposite side, and the adjacent side with respect to ∠B.



Example 2

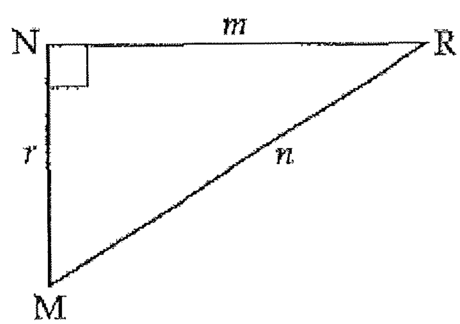
Identify the following sides in the triangle.



a) relative to ∠R.

opposite: r   adjacent: m   hypotenuse: n

b) relative to ∠M.

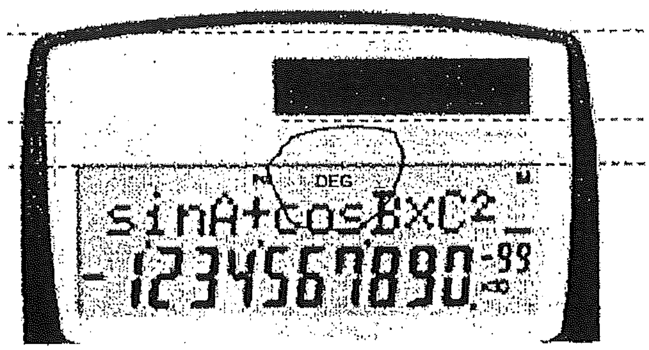


opposite: m   adjacent: r   hypotenuse: n

Scientific Calculator

There are three primary trigonometric ratios. You now can use the tangent ratio to solve for missing sides in a triangle. You will need a scientific calculator to perform trigonometric calculations. A scientific calculator has the tangent ratio key, abbreviated as "tan".

Angles can be measured in degrees, radians, or gradients. In this course, angles are measured only in degrees. Therefore, you must ensure that your calculator is always making trigonometric calculations using degrees. Your calculator must always be in the "DEG" mode, or sometimes "D" depending on the calculator. If it is in the "RAD" or "GRAD" mode, none of your answers will be correct. Always check that your calculator shows D or deg mode.



If your calculator is not showing degrees, it can be changed by using

- the mode key,
- or the DRG key,
- or sometimes by turning the calculator off and then on again,
- or pressing the reset button, depending on which calculator you have.

Finding the Tangent of an Angle

if you have a "backwards" calc, type in 60 $\boxed{tan}$

Example 1

Find the tangent of a 60° angle.

$\boxed{tan}$ 60

tan(60) = 1.732

Example 2

Find the tangent of 45°.

tan(45) = 1.

Example 3

Find the tangent ratio for each of these angles. Round off to 5 decimal places.

a) tan 50° =

1.19175

b) tan 22° =

0.40403

c) tan 38° =

0.78129

d) tan 16.8° =

0.30192

Inverse Tangent Process

Now that you understand how your calculator finds the tangent ratio of angles, you can do the entire process in reverse. You can find the angle given its tangent ratio.

You need to use the $\tan^{-1}$ key. This key is called "inverse tan".

To find the angle measurement given the tangent ratio, you need to find the key on your calculator that will access those small letters above the "tan" key. You may have to experiment with your calculator to find the right combination of key strokes. Usually you press SHIFT or 2nd and the tan key.

Example 1

The tangent ratio for an angle is 1.1918. Find the angle.

type $\boxed{2nd}$ $\boxed{tan}$ 1.1918

$\tan^{-1}(1.1918) = \underline{50^\circ}$

Example 2

Find each angle Θ, given tan Θ. Use the inv tan method you learned in Example 1 above. Round off your answer to the nearest degree.

Θ means angle.

a) $\tan \Theta = 0.70021$

$\theta = \tan^{-1}(0.70021) = \underline{35^\circ}$

b) $\tan \Theta = 2.35585$

$\theta = \tan^{-1}(2.35585) = \underline{67^\circ}$

c) $\tan \Theta = 0.14054$

$\theta = \tan^{-1}(0.14054) = \underline{8^\circ}$

Finding Values for Sides and Angles

When you use the tan ratio,

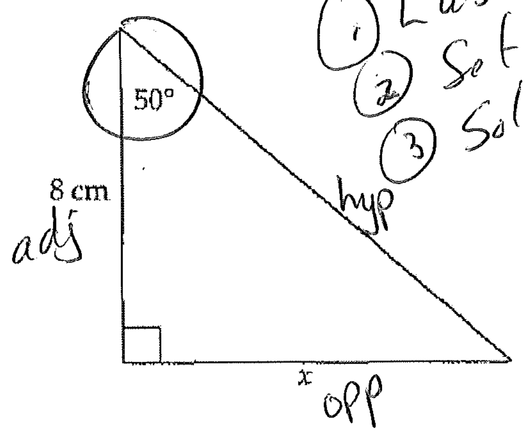
$$\tan \Theta = \frac{opposite}{adjacent},$$

there are three values, Θ, opposite, and adjacent that need to be considered. You would need to know any two of the values and then you can substitute these two known values into the formula to find the unknown value.

Example 1

Solve for the length of the side opposite the given angle.

1. Label
2. Set it up
3. Solve

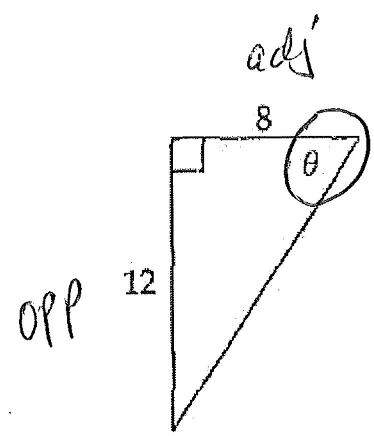


$$\frac{\tan 50}{1} = \frac{x}{8}$$

$$x = 9.53 \text{ cm}$$

Example 2

Solve for the angle.



$$\tan \theta = \left(\frac{12}{8}\right)$$

so

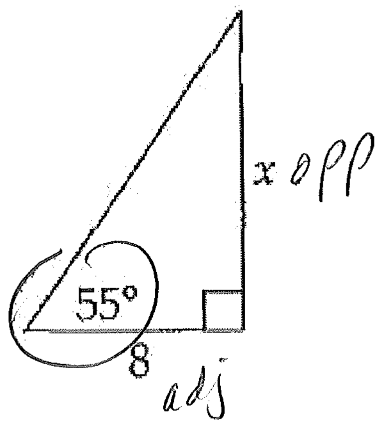
$$\theta = \tan^{-1}\left(\frac{12}{8}\right)$$

$$\theta = 56.3^\circ$$

PRACTICE QUESTIONS: TANGENT RATIO

1. Find the missing side using the tangent ratio. Round to two decimal places.

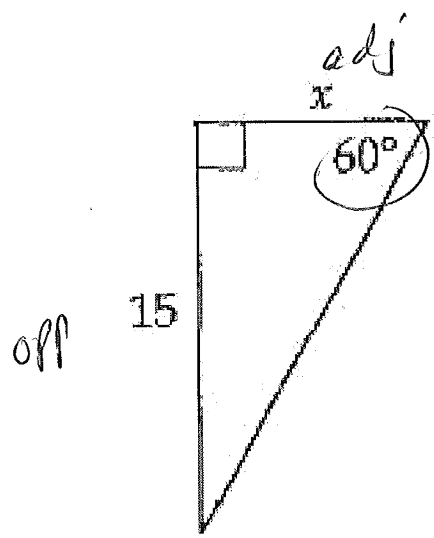
a)



$\frac{\tan 55}{1} = \frac{x}{8}$

$x = 11.43$

b)

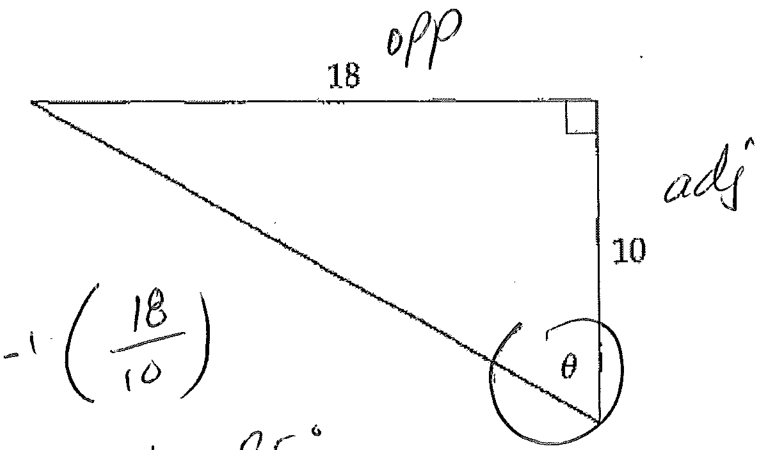


$\frac{\tan 60}{1} = \frac{15}{x}$

$x = 8.66$

2. Find the value of ϴ using the tangent ratio and its inverse. Round to two decimal places.

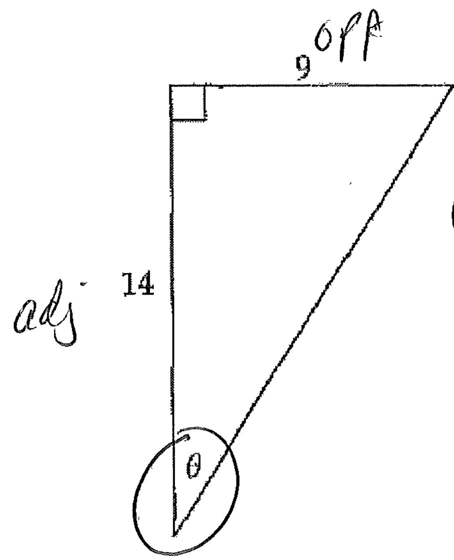
a)



$\theta = \tan^{-1}\left(\frac{18}{10}\right)$

$\theta = 60.95°$

b)



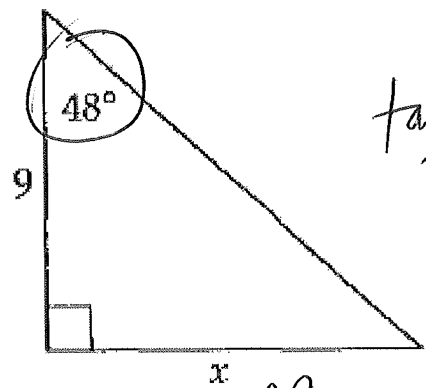
$\theta = \tan^{-1}\left(\frac{9}{14}\right)$

$\theta = 32.74°$

3. Use the tangent ratio to find the missing side. Round off your answer to two decimal places. Show your work.

a)

adj

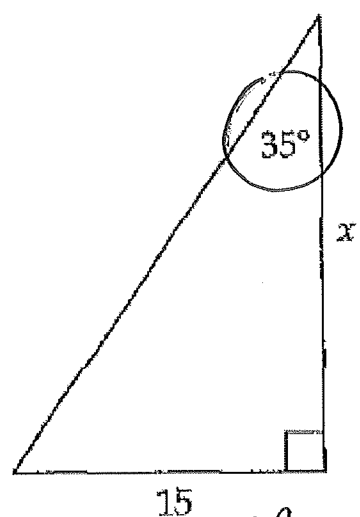


opp

$\frac{\tan 48}{1} = \frac{x}{9}$

$\underline{x = 10}$

b)



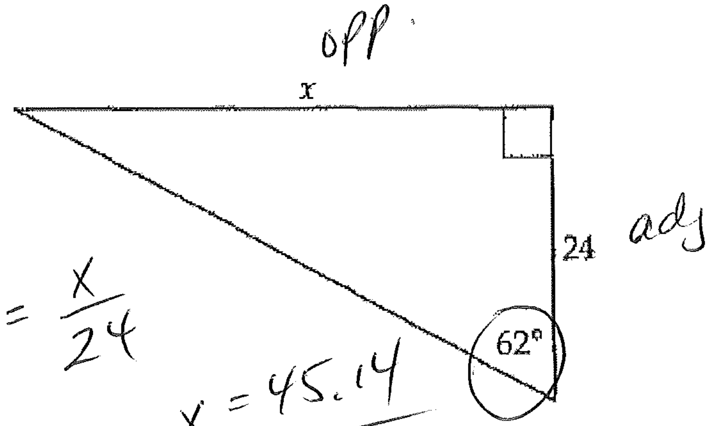
adj

opp

$\frac{\tan 35}{1} = \frac{15}{x}$

$x = \underline{21.42}$

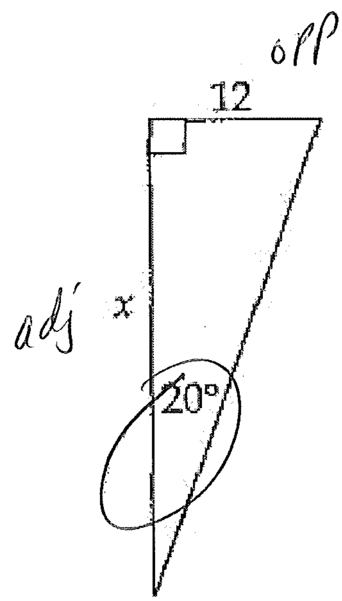
c)



$$\frac{\tan 62}{1} = \frac{x}{24}$$

$$x = \underline{45.14}$$

d)

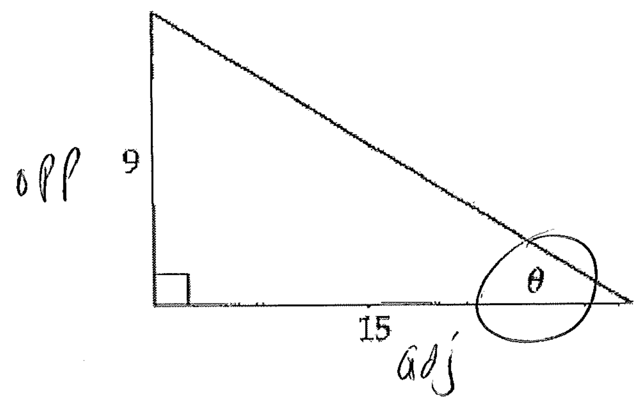


$$\frac{\tan 20}{1} = \frac{12}{x}$$

$$x = \underline{32.97}$$

4. Use the inverse tangent to find the missing angle. Show your work.

a)

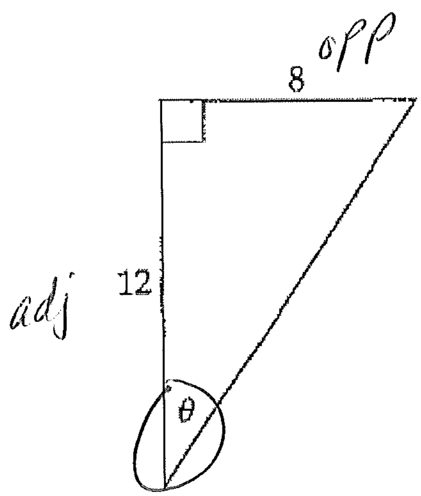


$\theta = \tan^{-1}\left(\frac{9}{15}\right)$

$\theta = 30.96°$

or $= 31°$

b)

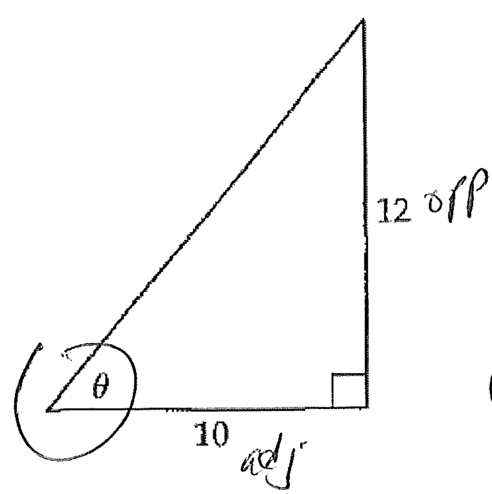


$\theta = \tan^{-1}\left(\frac{8}{12}\right)$

$\theta = 33.69°$

or $= 34°$

c)



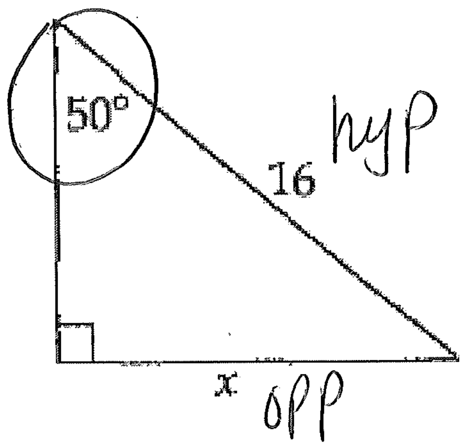
$\theta = \tan^{-1}\left(\frac{12}{10}\right)$

$\theta = 50.19°$

LESSON 5: SINE RATIO

The previous lesson worked exclusively with the tangent ratio. The sine ratio is very similar to the tangent ratio, except you now use the hypotenuse and the opposite side after designating a specific acute angle.

When you have or are looking for values for the opposite side and the hypotenuse, you use the sine ratio.



Notice that the x-side is opposite the given angle, and the side with a value of 16 is on the hypotenuse, the side opposite the right angle. The hypotenuse is always opposite the right angle.

When you are presented with a triangle using the opposite side and the hypotenuse, you use the sine ratio.

The tangent ratio is used when you have triangles with values on the opposite and adjacent sides.

When you use the sine ratio,

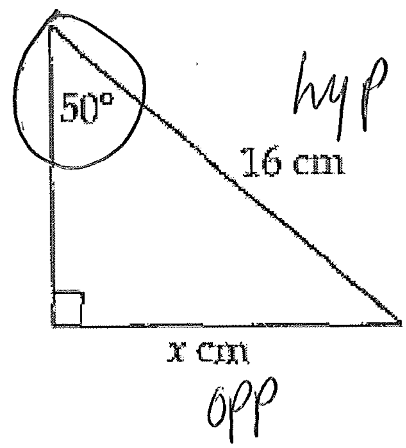
$$\sin \Theta = \frac{opposite}{hypotenuse},$$

you can substitute two known values into the formula. Then you can solve for the unknown value. The process is the same as the one you used in the previous lesson for the tangent ratio. Use the steps, multiply when the x is on the top, divide when x on the bottom, and use inverse sine when solving for an angle.

Finding Values for Sides

Example 1

Solve for x.



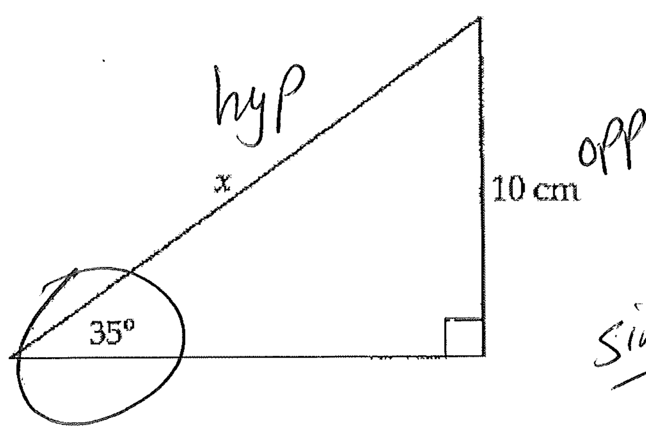
$\frac{\sin 50}{1} = \frac{x}{16}$

$x = \underline{12.26 \text{ cm}}$

or $x = \underline{12.3} \text{ cm}$

Example 2

Solve for x.



$\frac{\sin 35}{1} = \frac{10}{x}$

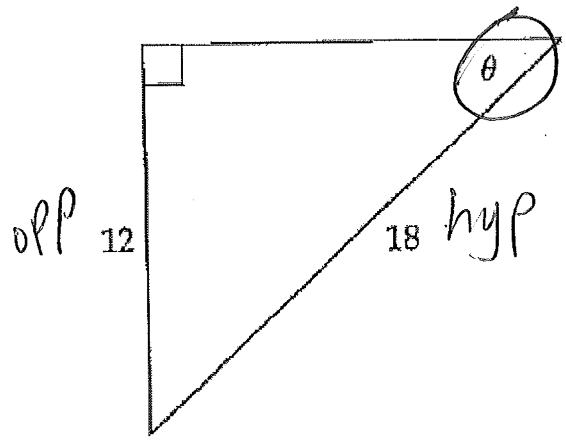
$x = \underline{17.43} \text{ cm}$

or $x = \underline{17.4 \text{ cm}}$

Finding Values for Angles

Example 1

Find the angle Θ.

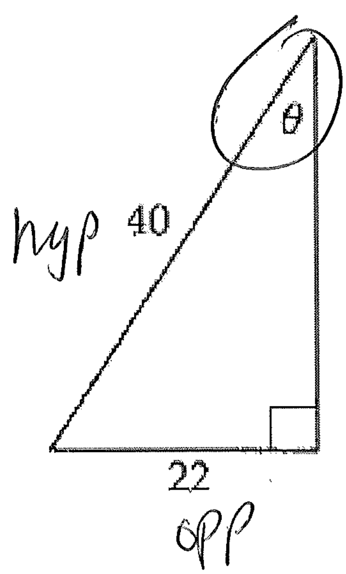


$\theta = \sin^{-1}\left(\frac{12}{18}\right)$

$\theta = \underline{\underline{41.8}}°$

Example 2

Find angle Θ.



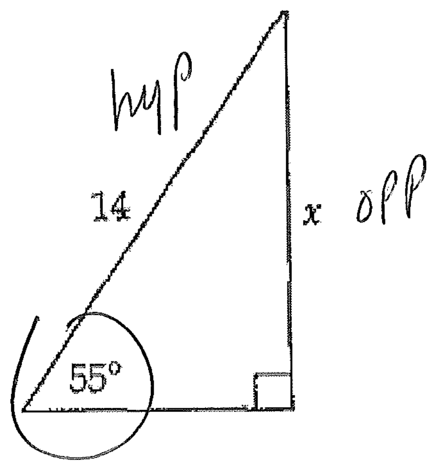
$\theta = \sin^{-1}\left(\frac{22}{40}\right)$

$\theta = \underline{33.4}°$

PRACTICE: SINE RATIO

1. Find the missing side using the sine ratio.

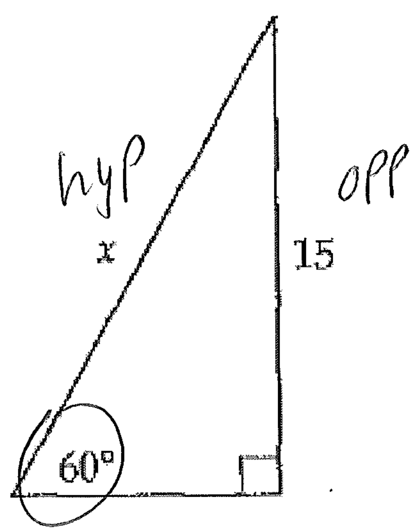
a)



$$\frac{\sin 55}{1} = \frac{x}{14}$$

$$x = 11.5$$

b)

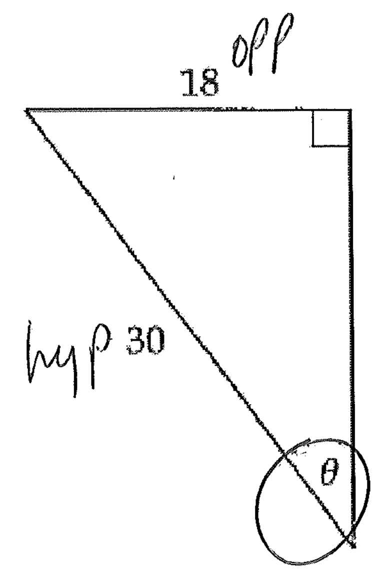


$$\frac{\sin 60}{1} = \frac{15}{x}$$

$$x = 17.3$$

2. Find angle Θ using the inverse sine.

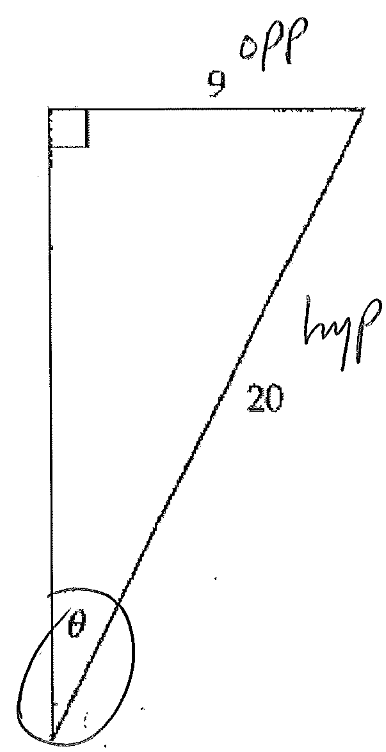
a)



$\theta = \sin^{-1}\left(\frac{18}{30}\right)$

$\theta = 36.9°$

b)

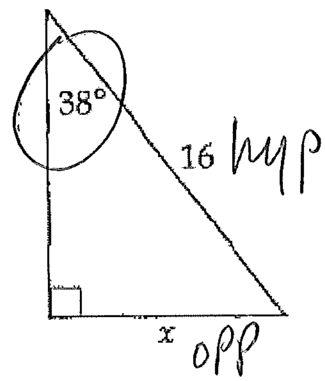


$\theta = \sin^{-1}\left(\frac{9}{20}\right)$

$\theta = 26.7°$

3. Use the sine ratio to find the missing side. Round your answer to two decimal places, if necessary. Show your work.

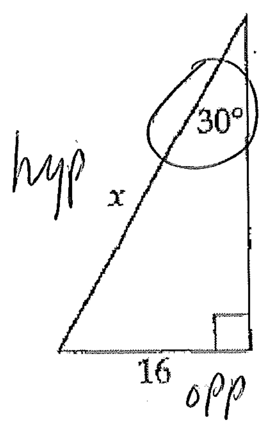
a)



$$\frac{\sin 38}{1} = \frac{x}{16}$$

$$x = \underline{\underline{9.9}}$$

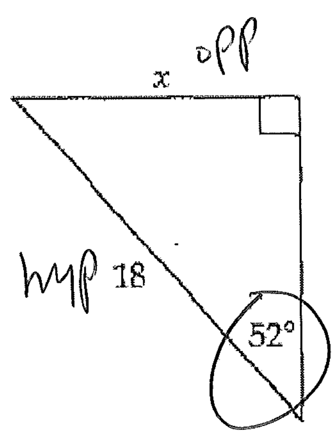
b)



$$\frac{\sin 30}{1} = \frac{16}{x}$$

$$x = \underline{32}$$

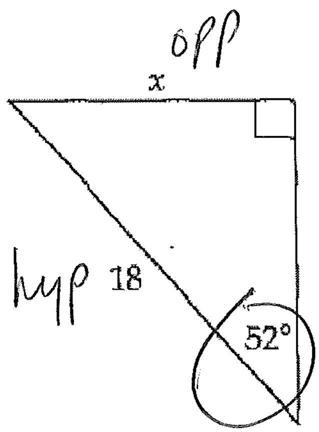
c)



$$\frac{\sin 52}{1} = \frac{x}{18}$$

$$x = \underline{14.2}$$

d)

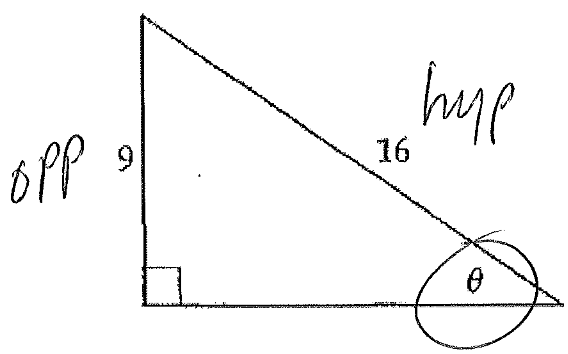


$$\frac{\sin 52}{1} = \frac{x}{18}$$

$$x = 14.2$$

4. Use the sine ratio and its inverse to find the missing angle. Round to two decimal places. Show your work.

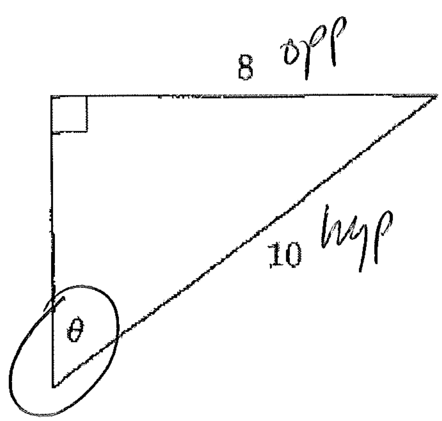
a)



$$\theta = \sin^{-1}\left(\frac{9}{16}\right)$$

$$\theta = 34.2^\circ$$

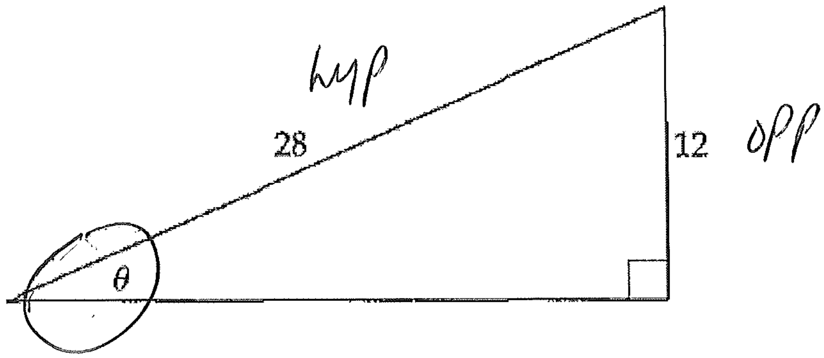
b)



$$\theta = \sin^{-1}\left(\frac{8}{10}\right)$$

$$\theta = 53.1^\circ$$

c)



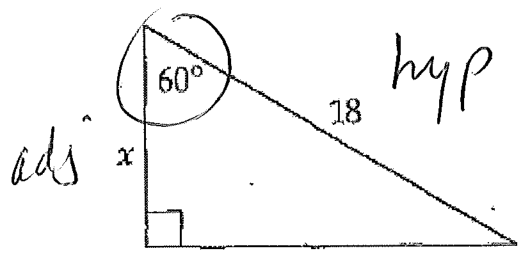
$$\theta = \sin^{-1}\left(\frac{12}{28}\right)$$

$$\theta = 25.4°$$

LESSON 6: COSINE RATIO

The third basic trigonometric ratio is the cosine. The cosine ratio is very similar to the tangent and sine ratios, except you now use the hypotenuse and the adjacent sides.

When you have or are looking for values for the adjacent side and the hypotenuse, you use the cosine ratio.



Notice that the x-side is adjacent to the given angle and the side with a value of 18 is on the hypotenuse. When you are presented with a triangle using the adjacent side and the hypotenuse, you use the cosine ratio. Remember that the tangent ratio is used when you have triangles with values on the opposite and adjacent sides, and the sine ratio is used when you have the opposite side and the hypotenuse.

Your scientific calculator will show the cosine key with the abbreviation "cos."

When you use the cosine ratio,

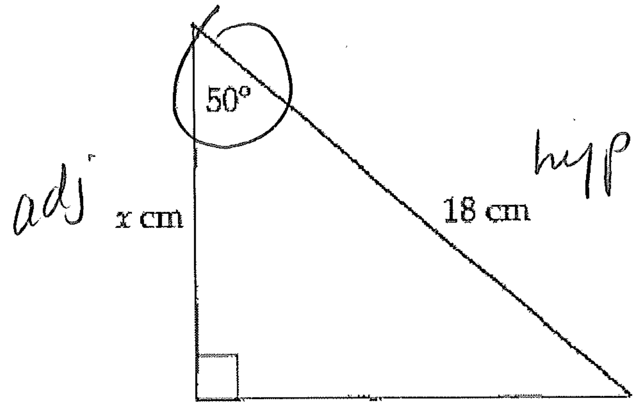
$$\cos \Theta = \frac{adjacent}{hypotenuse},$$

you can substitute two known values into the formula. Then you can solve for the unknown value. The process is the same as the one you used in the previous lessons for the tangent and sine ratios. Use the steps, multiply when the x is on the top, divide when x is on the bottom, and use inverse sine when solving for an angle.

Finding Values for Sides

Example 1

Solve for x.

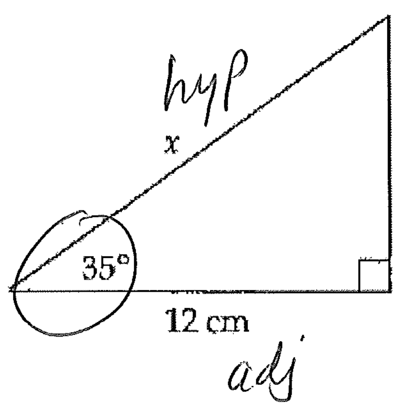


$\frac{\cos 50}{1} = \frac{x}{18}$

$x = \underline{11.6\text{ cm}}$

Example 2

Find the length of the hypotenuse.



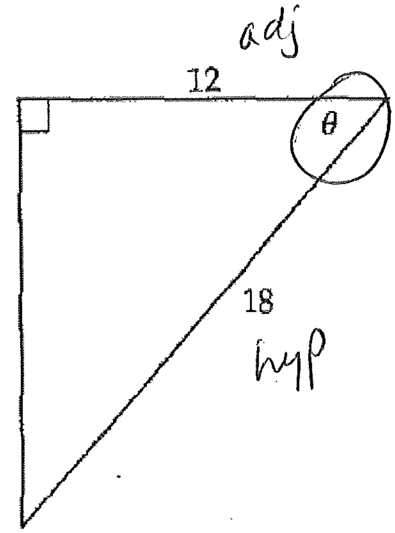
$\frac{\cos 35}{1} = \frac{12}{x}$

$x = \underline{14.7\text{ cm}}$

Finding Values for Angles

Example 1

You use your skills with finding the inverse of the cosine ratio to find the angle, Θ.



$\theta = \cos^{-1}\left(\frac{12}{18}\right)$

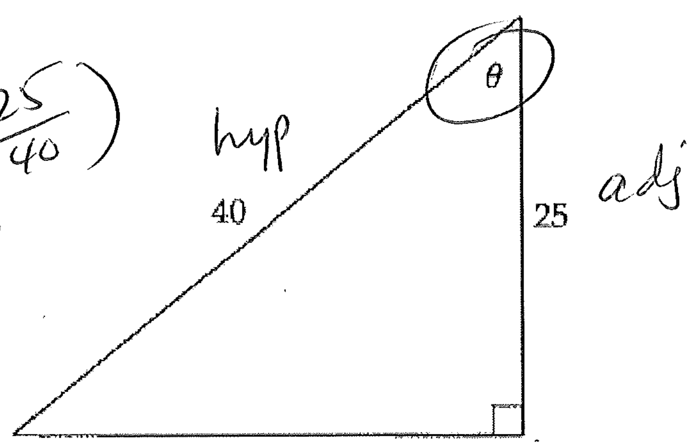
$\theta = \underline{48.2^{\circ}}$

Example 2

You use your skills with finding the inverse of the cosine ratio to find the angle Θ.

$\theta = \cos^{-1}\left(\frac{25}{40}\right)$

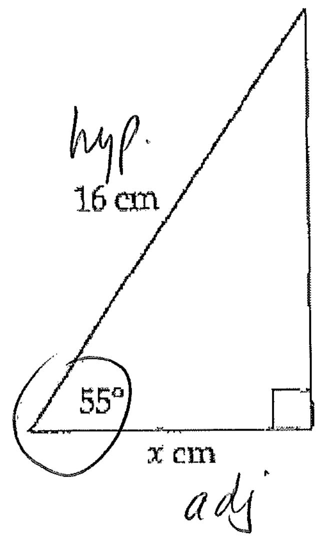
$= \underline{51.3^{\circ}}$



PRACTICE: COSINE RATIO

1. Find the missing side using the cosine ratio. Round off to two decimal places.

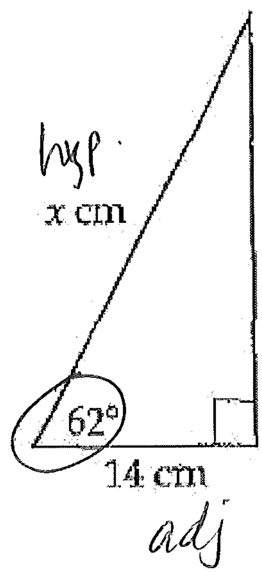
a)



$$\frac{\cos 55}{1} = \frac{x}{16}$$

$$x = 9.2 \text{ cm}$$

b)

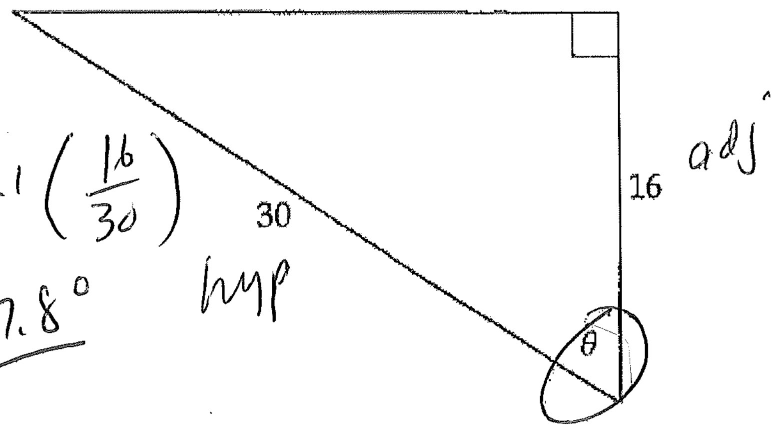


$$\frac{\cos 62}{1} = \frac{14}{x}$$

$$x = 29.8 \text{ cm}$$

2. Find angle Θ using the cosine ratio and its inverse.

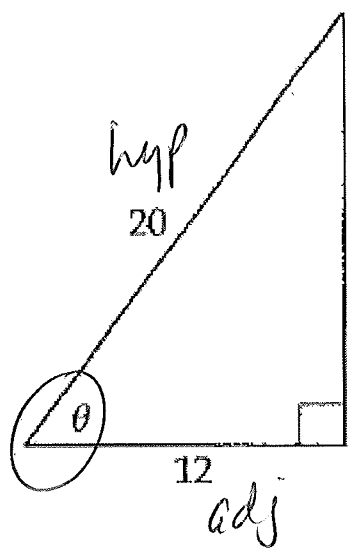
a)



$\theta = \cos^{-1}\left(\frac{16}{30}\right)$

$\theta = \underline{57.8^\circ}$

b)



$\theta = \cos^{-1}\left(\frac{12}{20}\right)$

$\theta = \underline{53.1^\circ}$

SOH CAH TOA

Three ratios, each with a different arrangement of sides, can be difficult to remember. A tool you can use is SOH CAH TOA. The letters in each group tell you which ratio to use, depending on which sides are involved in the question.

SOH represents Sin ϴ = $\frac{opposite}{hypotenuse}$

CAH represents Cos ϴ = $\frac{adjacent}{hypotenuse}$

TOA represents Tan ϴ = $\frac{opposite}{adjacent}$

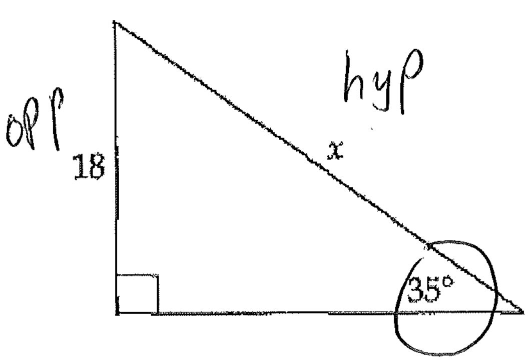
Add to your study sheet.

Selecting the Ratio

When you are presented with a right triangle to solve, you need to decide which of the three ratios you can use to solve for the unknown.

Example 1

Solve for x.



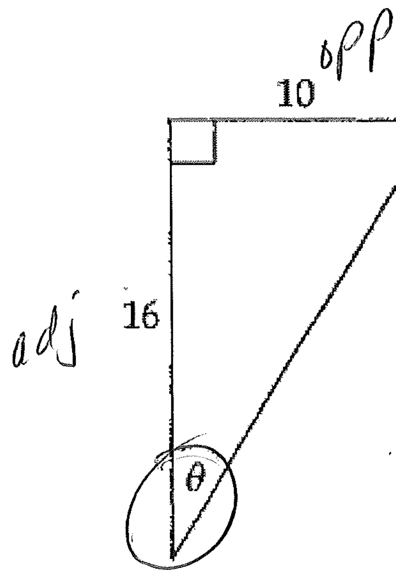
opp, hyp → sine

$$\frac{\sin 35}{1} = \frac{18}{x}$$

$$x = 31.4$$

Example 2

Find the measure of Θ.



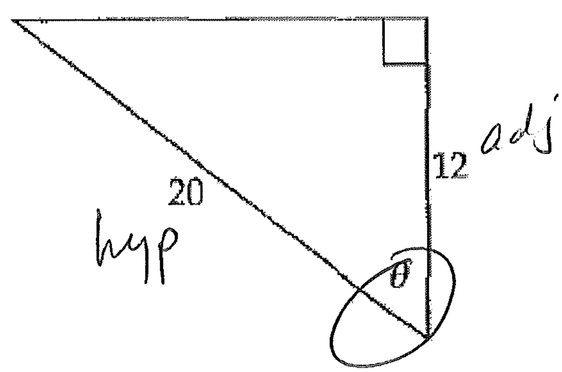
opp, adj → tan

$\theta = \tan^{-1}\left(\frac{10}{16}\right)$

$\theta = 32°$

Example 3

Find the measure of Θ.



adj, hyp ⇒ cos

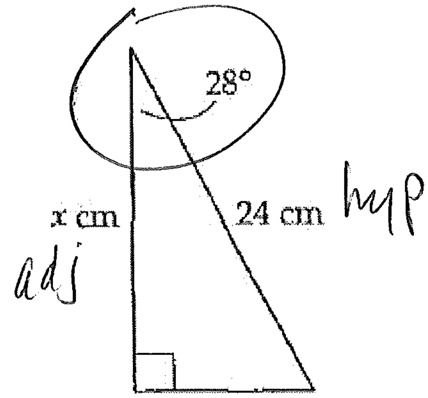
$\theta = \cos^{-1}\left(\frac{12}{20}\right)$

$\theta = 53.1°$

PRACTICE: TRIGONOMETRIC RATIOS

1. Use one of the three primary trigonometry ratios to solve for each unknown. Round to two decimal places.

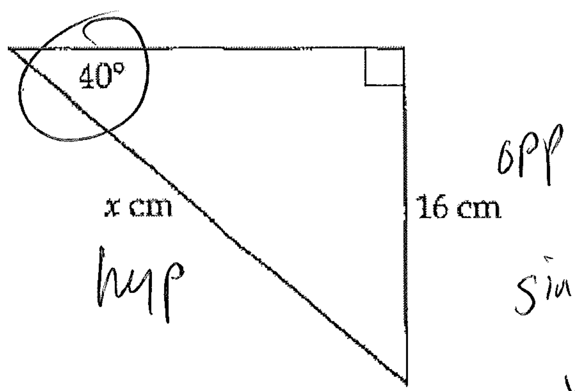
a)



$$\frac{\cos 28}{1} = \frac{x}{24}$$

$x = 21.2$ cm

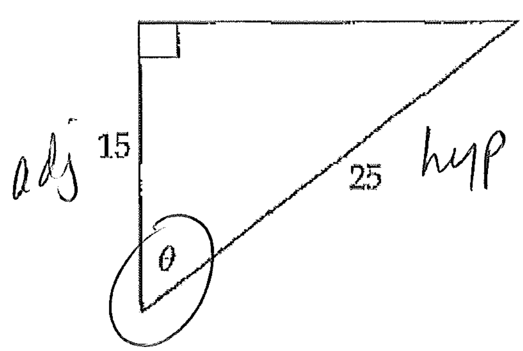
b)



$$\sin 40 = \frac{16}{x}$$

$x = 24.9$ cm

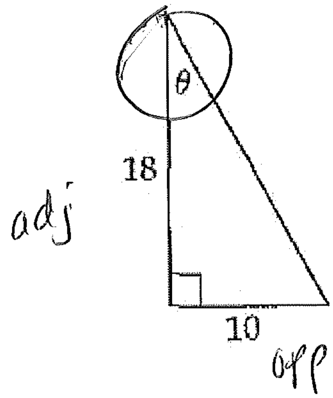
c)



$$\theta = \cos^{-1}\left(\frac{15}{25}\right)$$

$\theta = 53.1°$

d)

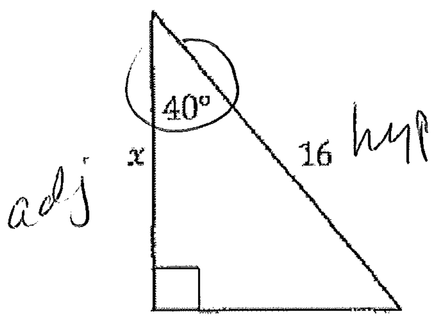


$$\theta = \tan^{-1}\left(\frac{10}{18}\right)$$

$$\theta = 29.1^\circ$$

2. Use trigonometry ratios to find the indicated side. Round to two decimal places. Show your work.

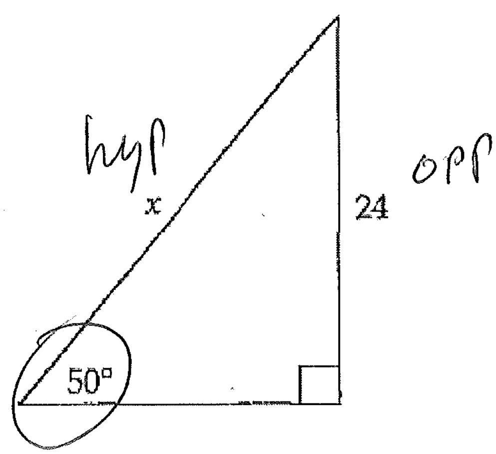
a)



$$\frac{\cos 40}{1} = \frac{x}{16}$$

$$x = 12.3$$

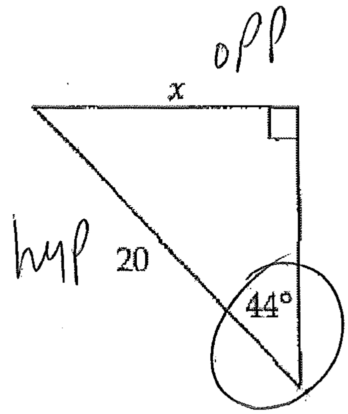
b)



$$\frac{\sin 50}{1} = \frac{24}{x}$$

$$x = 31.3$$

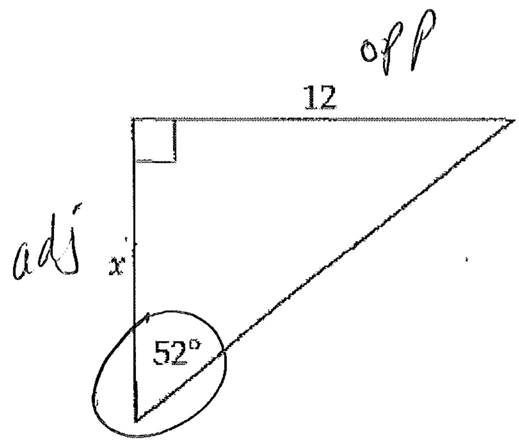
c)



$$\frac{\sin 44}{1} = \frac{x}{20}$$

$$x = 13.9$$

d)

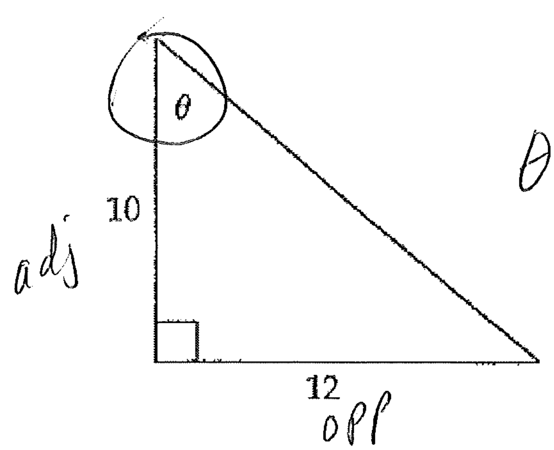


$$\frac{\tan 52}{1} = \frac{12}{x}$$

$$x = 9.4$$

3. Use trigonometry ratios and their inverses to find the indicated angles. Round your answers to two decimal places. Show your work.

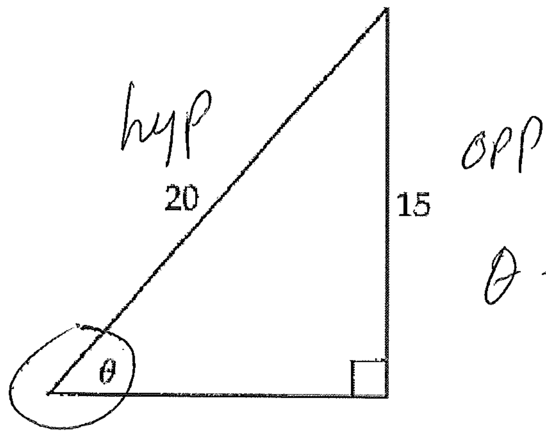
a)



$$\theta = \tan^{-1}\left(\frac{12}{10}\right)$$

$$\theta = 50.2^\circ$$

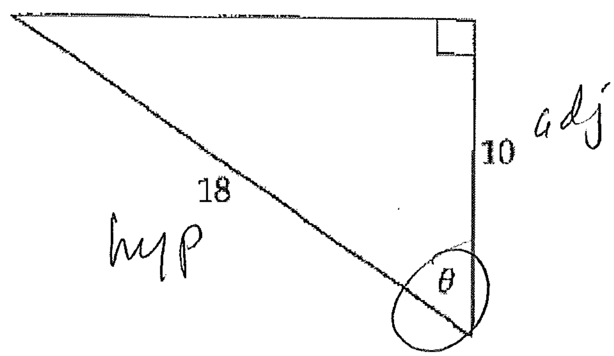
b)



$\theta = \sin^{-1}\left(\frac{15}{20}\right)$

$\theta = 48.6°$

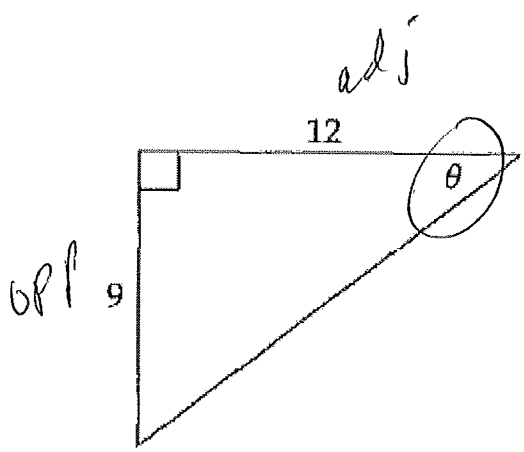
c)



$\theta = \cos^{-1}\left(\frac{10}{18}\right)$

$\theta = 56.3°$

d)



$\theta = \tan^{-1}\left(\frac{9}{12}\right)$

$\theta = 36.9°$